Symmetry breaking of QCD in a strong magnetic field

Yoshimasa Hidaka (RIKEN)

Based on Kenji Fukushima, YH, Phys. Rev. Lett 110, 031601 (2013), arXiv:1209.1319
YH, Arata Yamamoto, 1209.0007

Orders of magnitude for magnetic fields

wikipedia

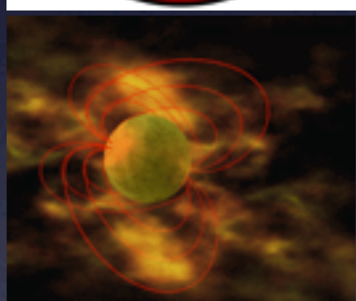
Typical magnet 50G

Neodymium magnet 12,500G

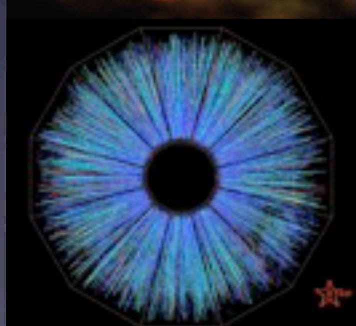
(strongest permanent magnet)

Strongest continuous magnetic field 450,000G

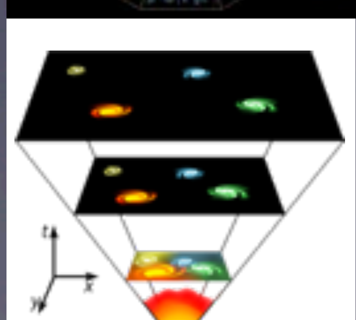
produced in a laboratory



Magnetars $\sim 10^{13}$ G



Heavy ion collisions $\sim 10^{4}\mathrm{MeV}^{2} \sim 10^{17}$ G



The early Universe $\sim 10^{22}$ G

(Electroweak transition)

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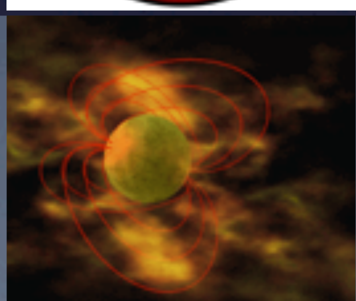

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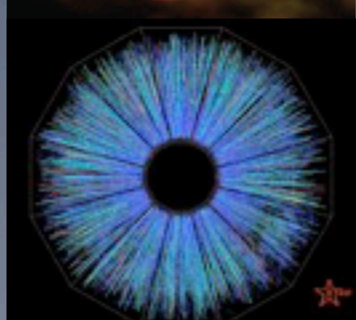

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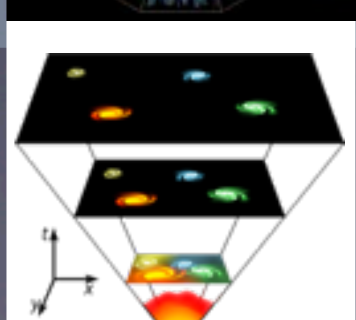


The early Universe $\sim 10^{22}$ G

(Electroweak transition)

Strong magnetic field in heavy ion collisions

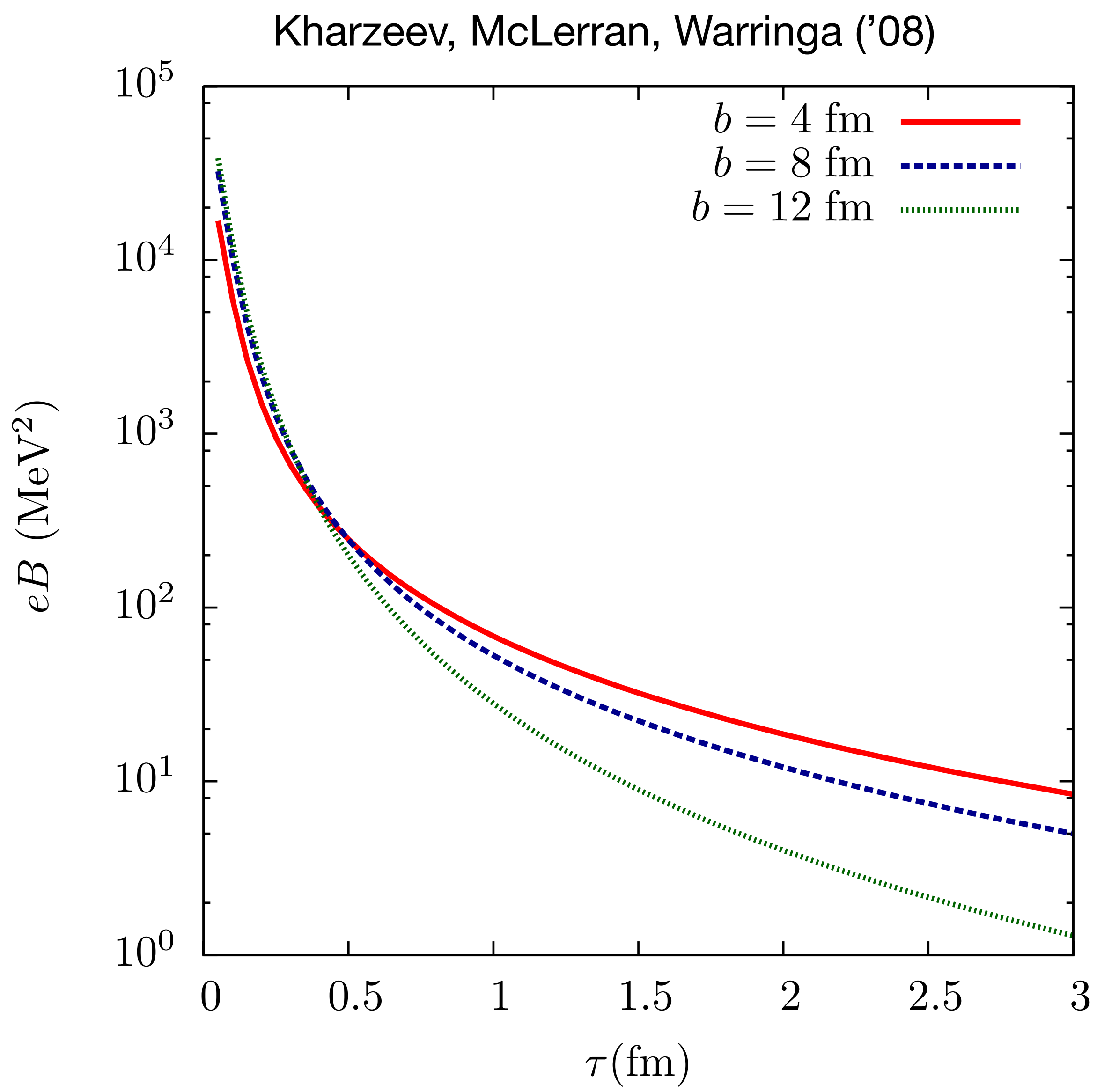
Strong magnetic field in heavy ion collisions

Strong magnetic field $\sqrt{eB} \sim 100\mathrm{MeV}$

$\sim 10^{17} - 10^{18}\mathrm{Gauss}$

Kharzeev, McLerran, Warringa (2008)

Magnetic field in heavy ion collisions



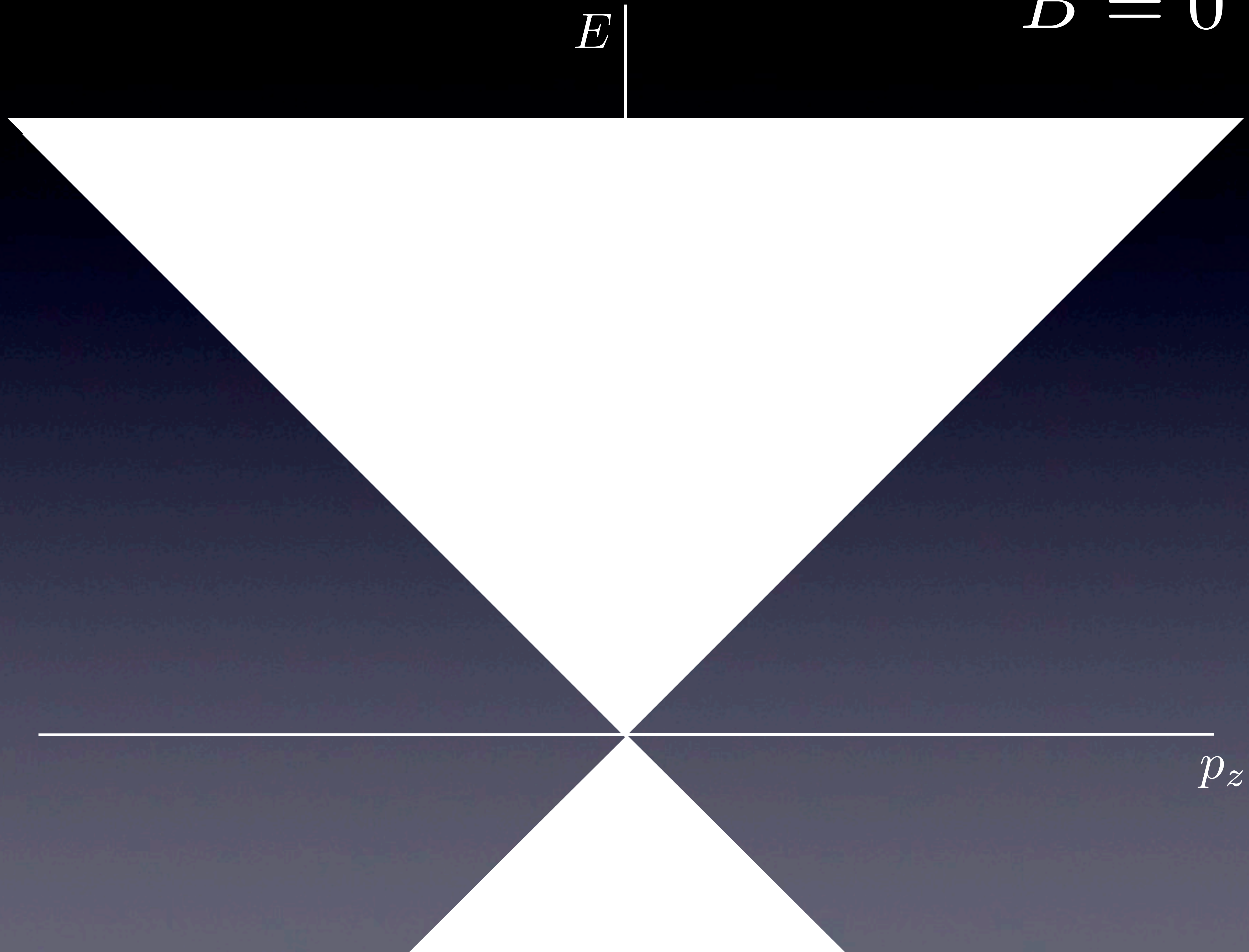
Strong magnetic filed is the QCD scale.

Part I: Chiral symmetry breaking

Part II: Fate of Vector meson

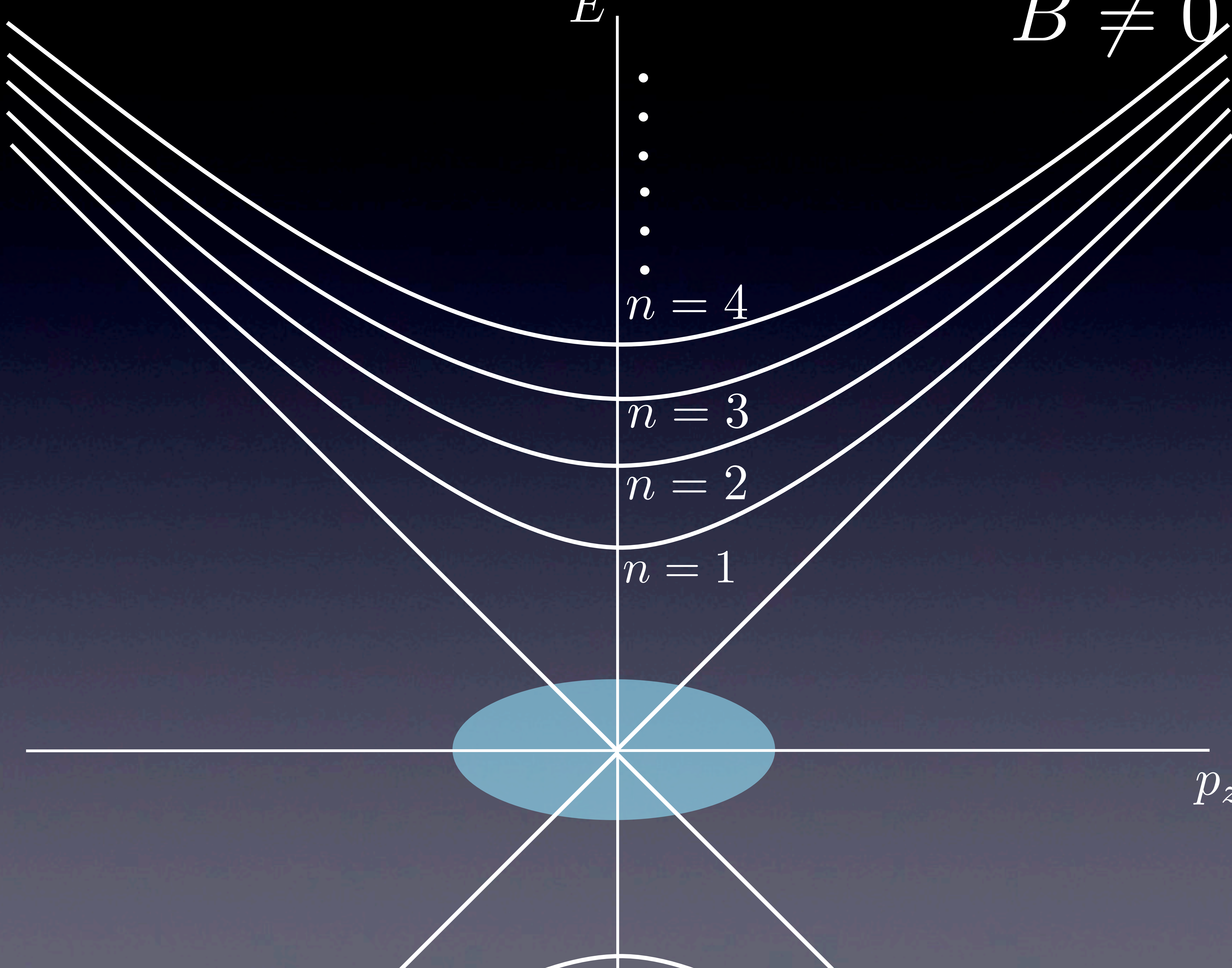
Part I:
Magnetic catalysis
vs
Magnetic Inhibition

Dirac Spectrum $m = 0$ $B = 0$



Dirac Spectrum

$m = 0$
$B \neq 0$



Magnetic Catalysis

Gusynin, Miransky, Shovkovy('94), Review: Shovkovy, 1207.5081

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Landau quantization:

1+1 dimension

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⬇

Particle-hole instability (LLL)

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Landau quantization: **1+1 dimension**

↓

Particle-hole instability (LLL)

↓

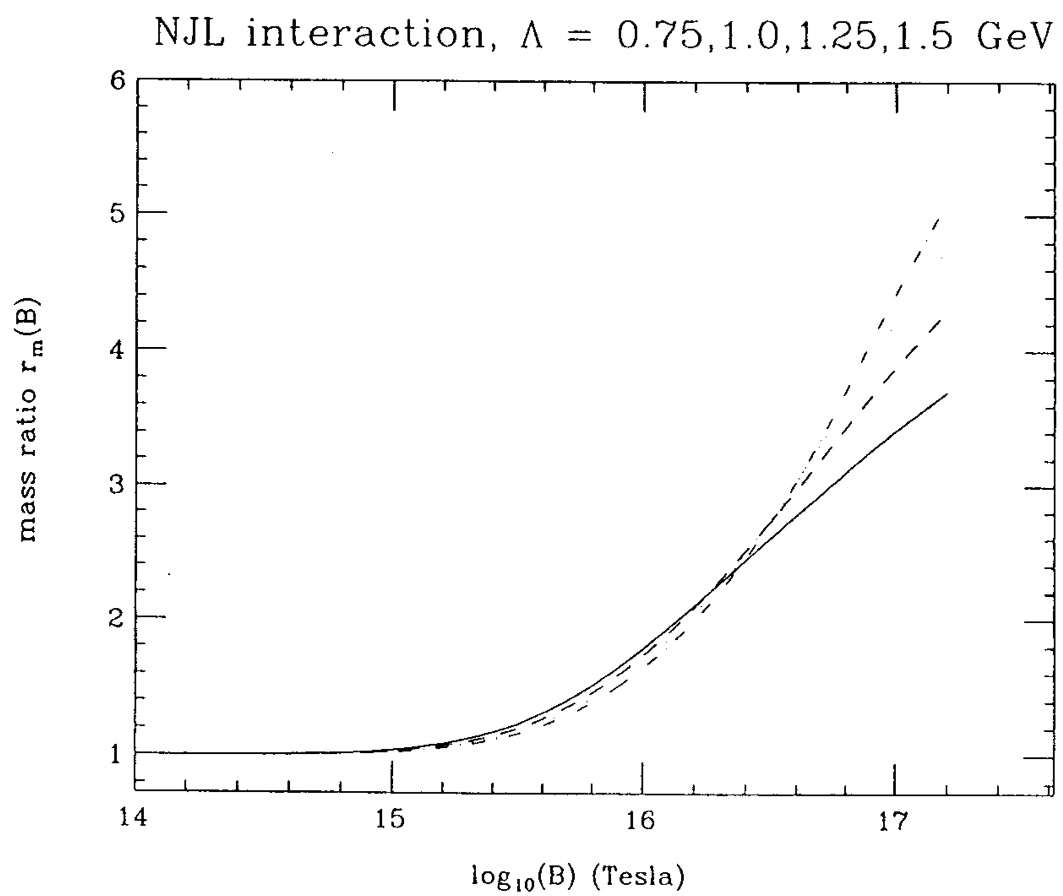
Spontaneous breaking of chiral symmetry

Model and Lattice Study T=0

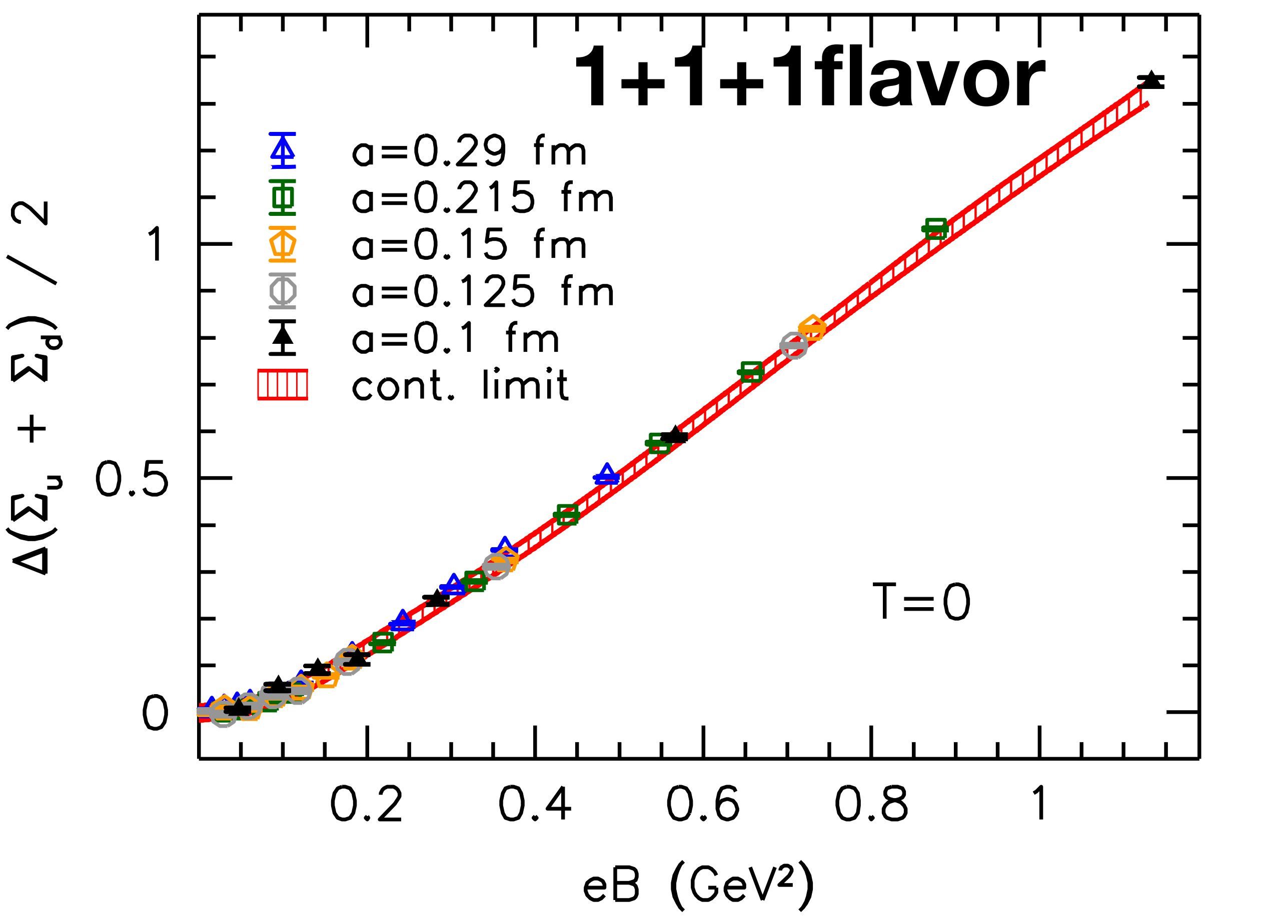
Kawati, Konishi, Miyata('83), Klevansky, Lemmer('89),
Suganuma, Tatsumi('91), Klimenko('92)
Krive, Naftulin('92), Schramm, Muller, Schramm('92)

Buividovich, Chernodub, Luschevskaya, Polikarpov ('09)
Braguta, Buividovich, Kalaydzhyan, Kuznetsov, Polikarpov('10)
D'Elia, Mukherjee, Sanfilippo('10) M. D'Elia and F. Negro('11)
Ilgenfritz, M. Kalinowski, M. Muller-Preussker, B. Petersson, and A. Schreiber('12)

Schramm, Muller, Schramm Mod. Phys. Lett. A 07, 973 (1992)



Bali, Bruckmann, Endrodi, Fodor, Katz, Schafer, JHEP 1202 (2012) 044



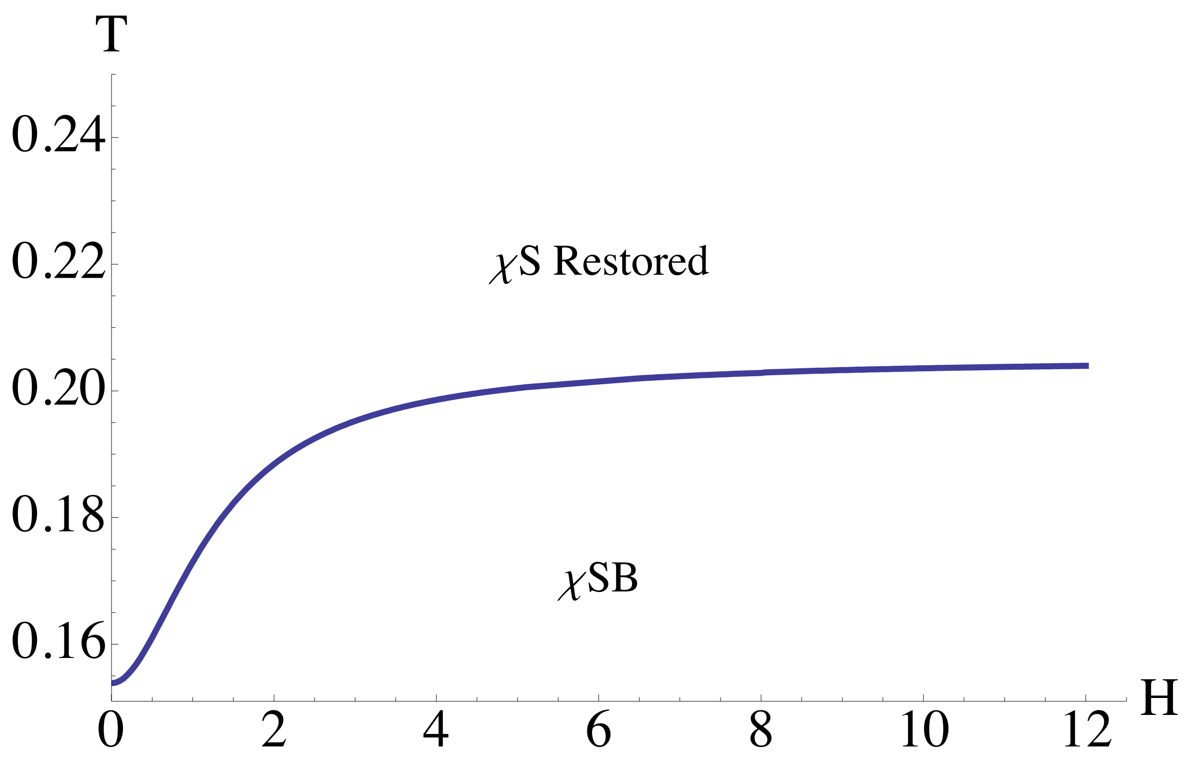
Condensate becomes large.

Model Study T≠0

Johnson, Kundu('08), Fraga, Mizher('09). Mizher, Chernodub, Fraga('10), Fukushima, Ruggieri, Gatto('10), Gatto, Ruggieri('10) ('11), Skokov('11), Fukushima, Pawlowski ('12), Andersen, Tranberg ('12)

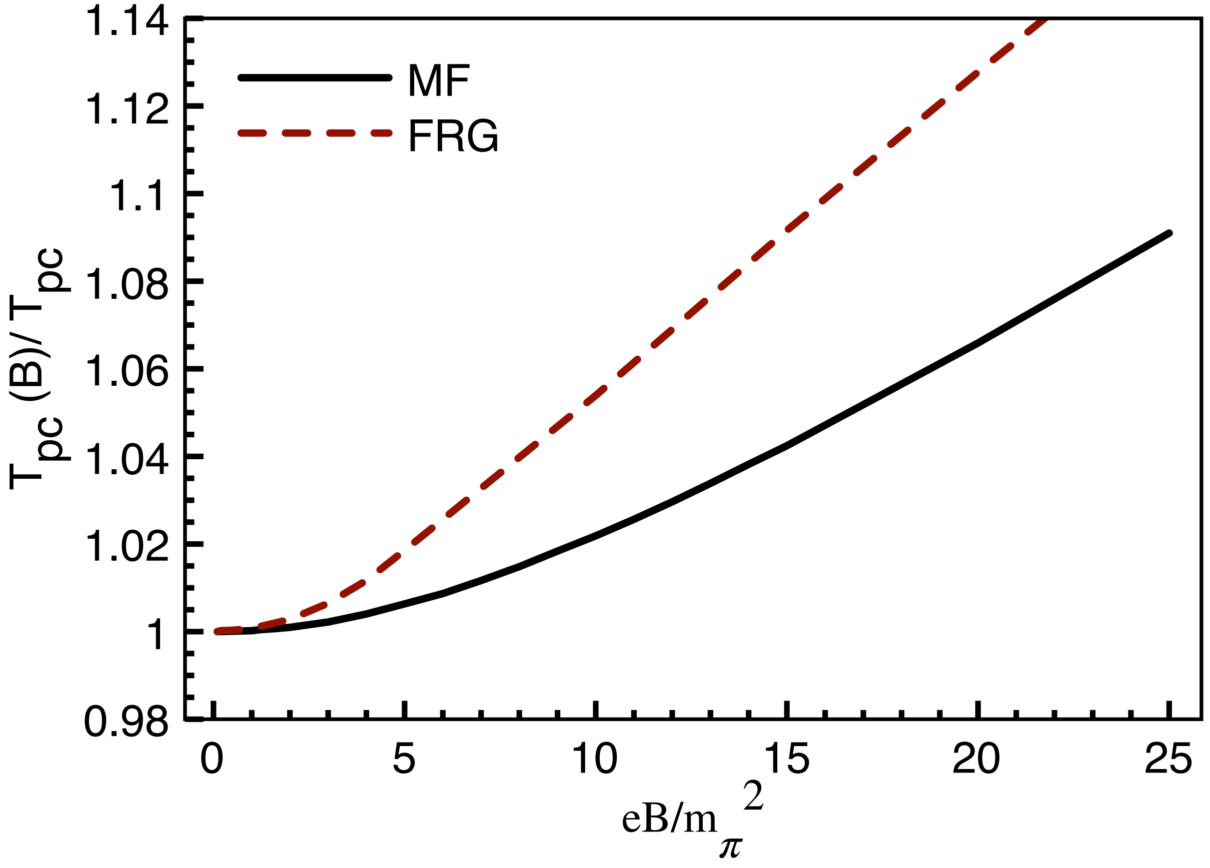
Sakai-Sugimoto

Johnson, Kundu, JHEP12 (2008) 053



PQM

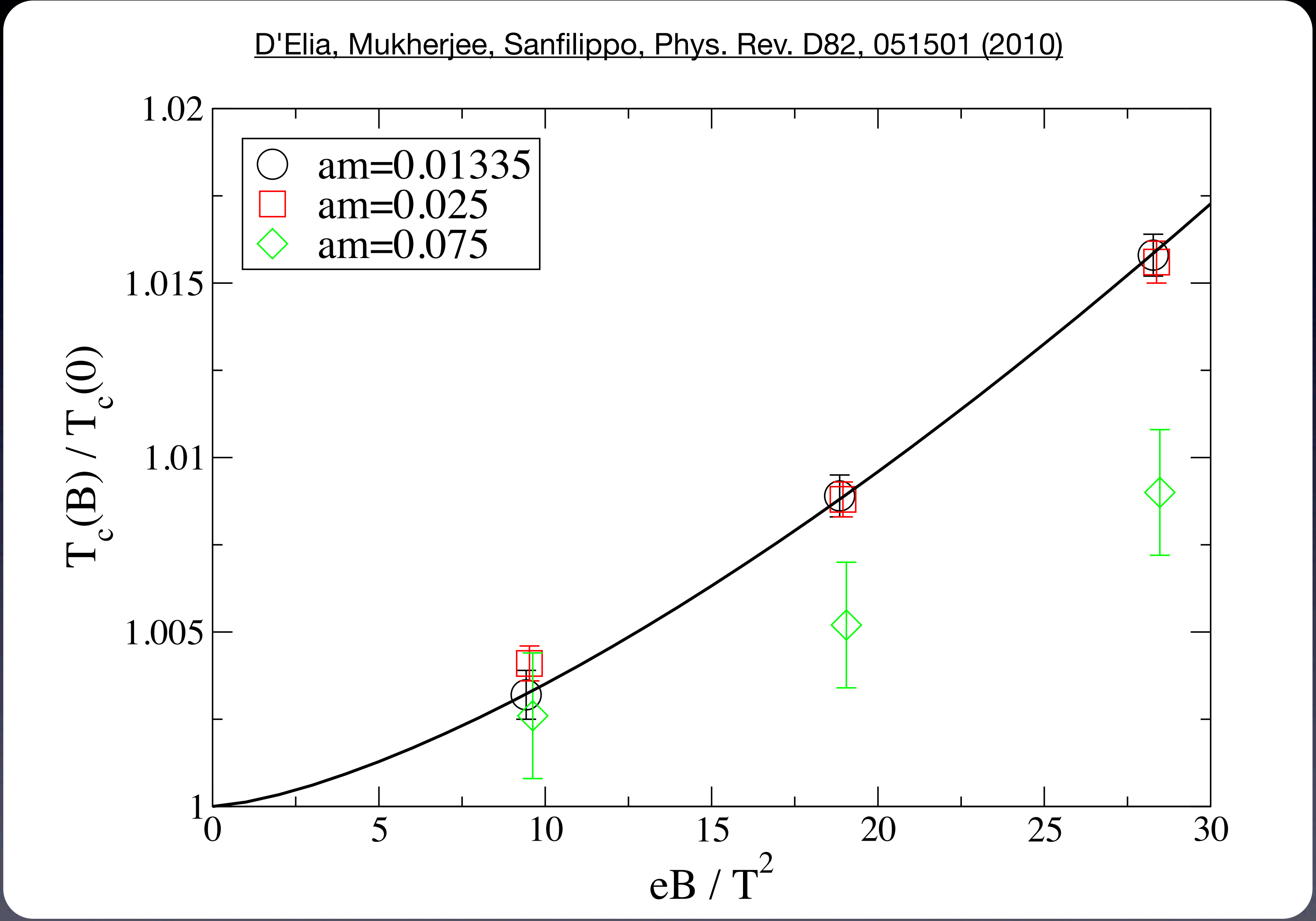
Skokov, 1112.5137



Lattice Study $T \neq 0$

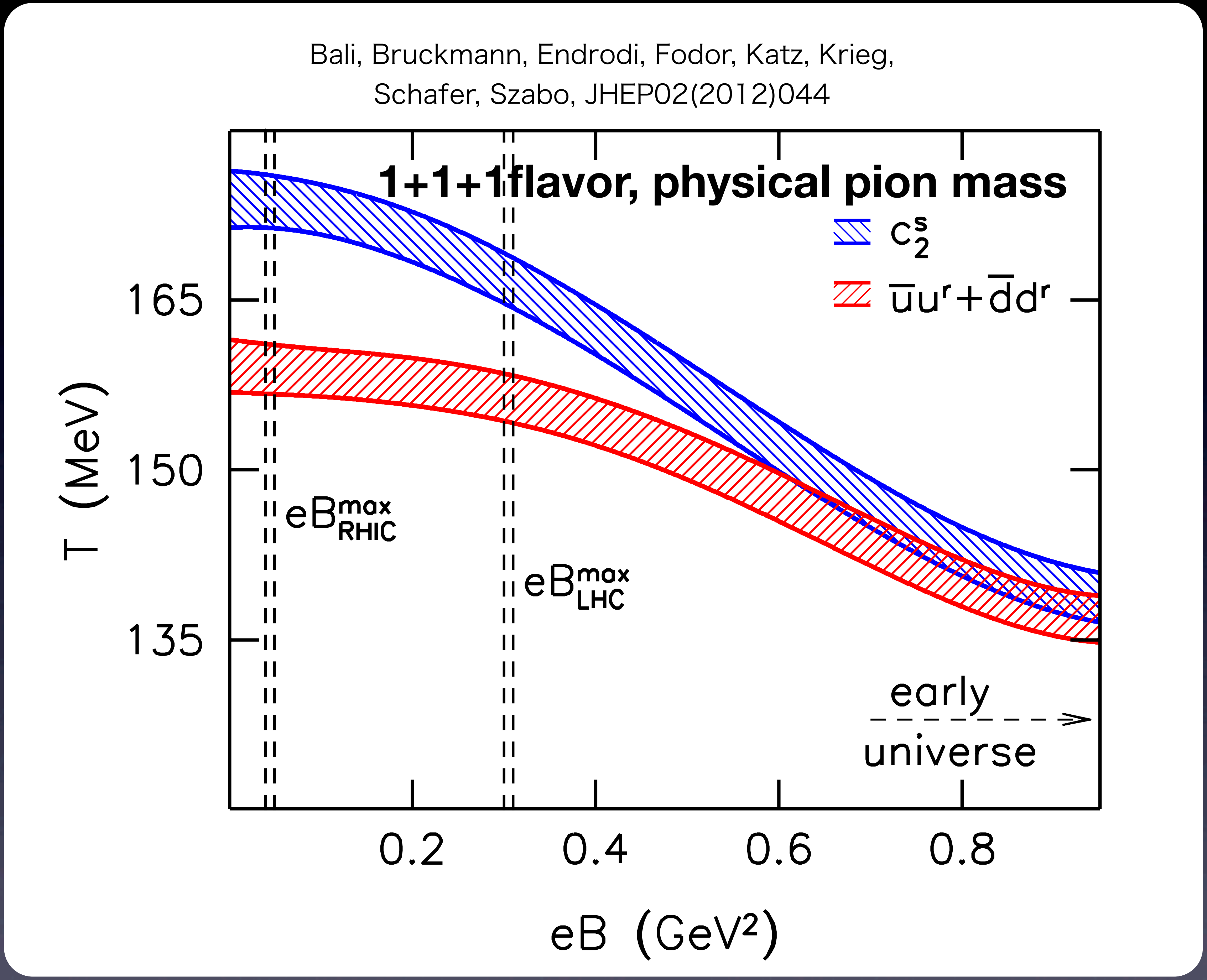
D'Elia, Mukherjee, Sanfilippo('10) M. D'Elia and F. Negro('11)

Two color: Ilgenfritz, M. Kalinowski, M. Muller-Preussker, B. Petersson, and A. Schreiber('12)



T_c increases as B increases.

Recent Lattice result



Decreasing T_c!

“Inverse Magnetic Catalysis”

Why does the Inverse Magnetic Catalysis occurs?

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Our possible answer:

Magnetic inhibition

Fukushima, YH ('12)

Mermin-Wagner's theorem

Mermin, Wargner ('66), Hohenberg ('67), Coleman ('73)

In Quantum 1+1d, classical 2+1d(at finite-T), continuous symmetries cannot be spontaneously broken

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In Quantum 1+1d, classical 2+1d(at finite-T), continuous symmetries cannot be spontaneously broken

Quarks:1+1d like because of Landau quantization

However, neutral pions: 3+1d like, so OK?

Model study

Fukushima, YH ('12)

NJL model

$$\mathcal{L} = \bar{\psi} i \not{D} \psi + \frac{G}{2} \left[(\bar{\psi}\psi)^2 + (\bar{\psi} i\gamma_5 \psi)^2 \right]$$

$$\not{D} \equiv \gamma^\mu (\partial_\mu + ieA_\mu)$$

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Quark contribution (LLL)

$$V_{\mathrm{q}}(\sigma) \simeq \frac{eB}{8\pi^2}\left(\Lambda^2 - \sigma^2 \ln\frac{e^{1-\gamma}\Lambda^2}{\sigma^2}\right).$$

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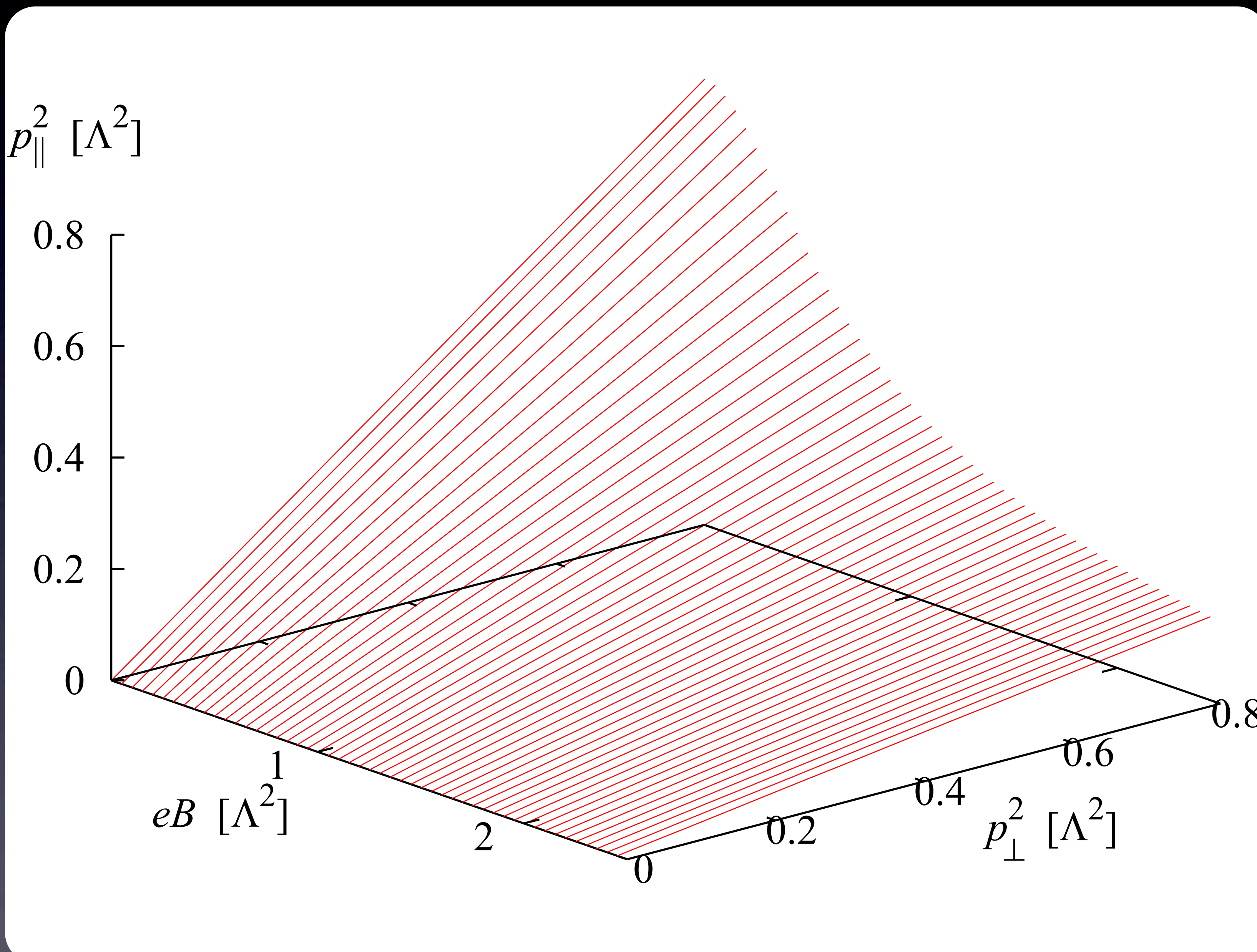
Consider the pion contribution $V_\pi(\sigma)$

Dispersion relation of pions

Fukushima, YH ('12)



Velocity of pion for the transverse direction becomes small.

$$E^2 \simeq p_z^2 + v_\perp^2 p_\perp^2$$

looks like 1+1d

$$v_\perp^2 \sim \frac{\sigma^2}{eB}$$

for $p_\perp^2 < eB$

Potential

Quark contribution

$$V_{\mathrm{q}}(m) \simeq -\frac{eB}{8\pi^2}\sigma^2 \ln \frac{e^{1-\gamma}\Lambda^2}{\sigma^2}$$

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$$V_\pi(m) \simeq \xi \frac{eB}{16\pi^2}\sigma^2 \ln\left[\frac{e^{2-\gamma}\Lambda_\pi^2}{\sigma^2}\right]$$

Magnetic Inhibition

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Magnetic Inhibition

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Need more quantitate analysis.

At finite temperature

Vanishing Magnetic Catalysis

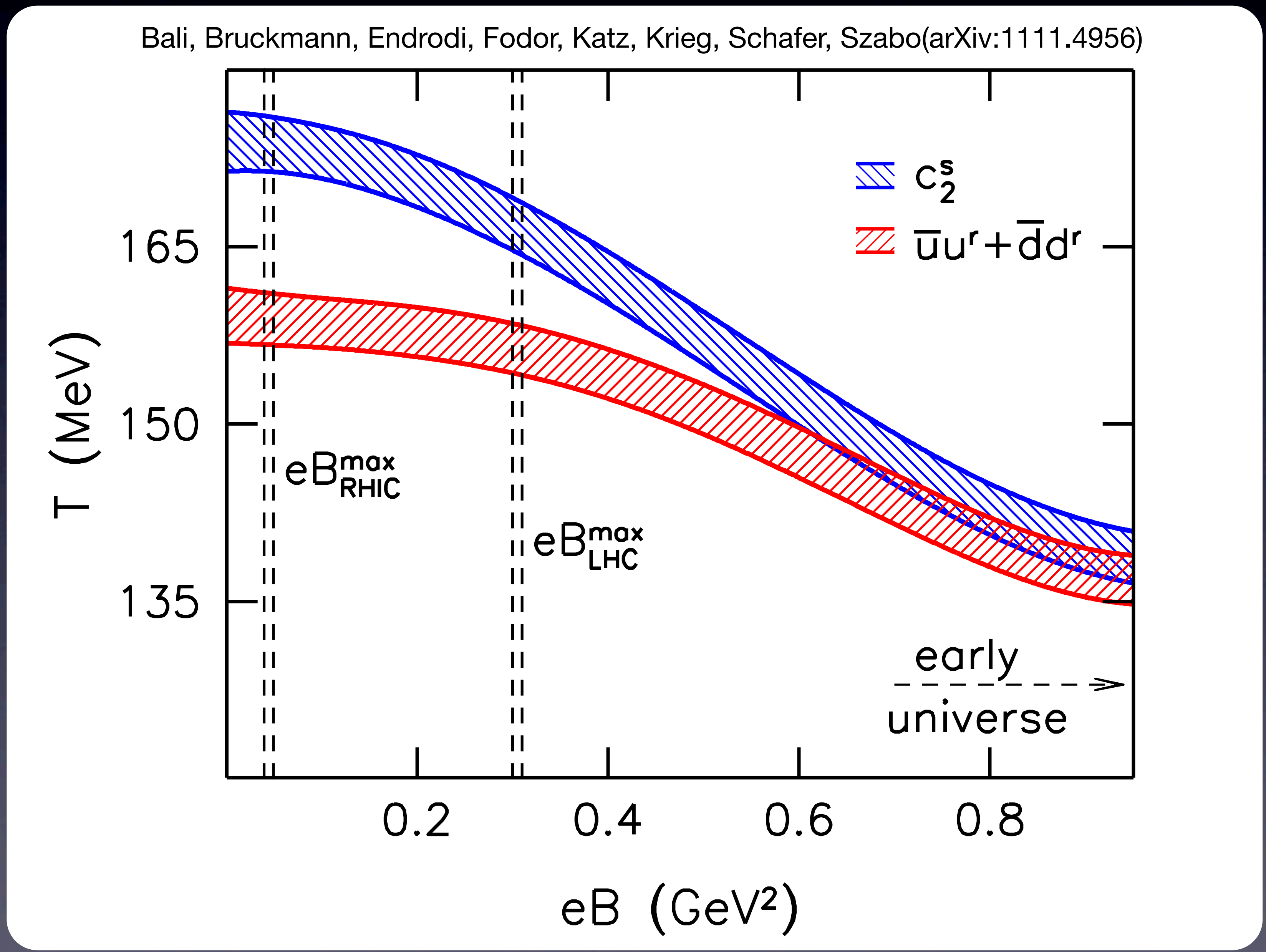
Dirac surface is smeared.

Lowest Matsubara mode plays a role of the infrared cutoff:

$$D_q \sim \frac{1}{T^2 + p^2} \neq 0$$

The infrared singularity vanishes.

At finite temperature
Vanishing Magnetic Catalysis
Pion fluctuations get stronger.



Consistent with the recent Lattice calculation

Part I: Summary

Magnetic Inhibition

Pion dispersion: $E^2 \simeq p_z^2 + v_\perp^2 p_\perp^2 \quad v_\perp^2 \sim \dfrac{\sigma^2}{eB}$

Transverse velocity becomes small as B increases.

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Vacuum: Need more quantitative analysis.

Finite temperature:

Fermi-distribution: Magnetic catalysis becomes weaker.

Bose-distribution: Magnetic Inhibition becomes stronger.

Decreasing T_C is consistent with Lattice result.

Part II:
Fate of vector meson

Vector meson

$$m_\rho^2(B) \approx m_\rho^2 - eB$$

$$m_\rho^2(B = B_c) = 0$$

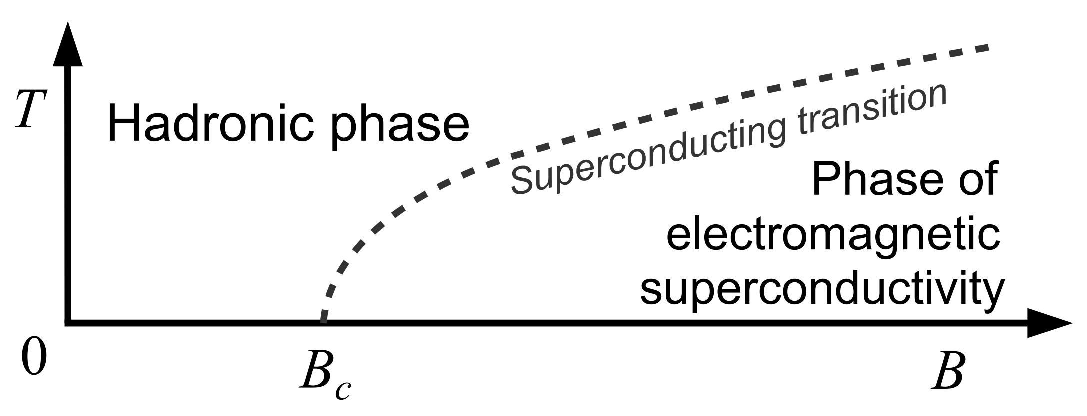
Vector meson condensation?

Schramm, Muller, and Schramm ('92)

Model analysis:

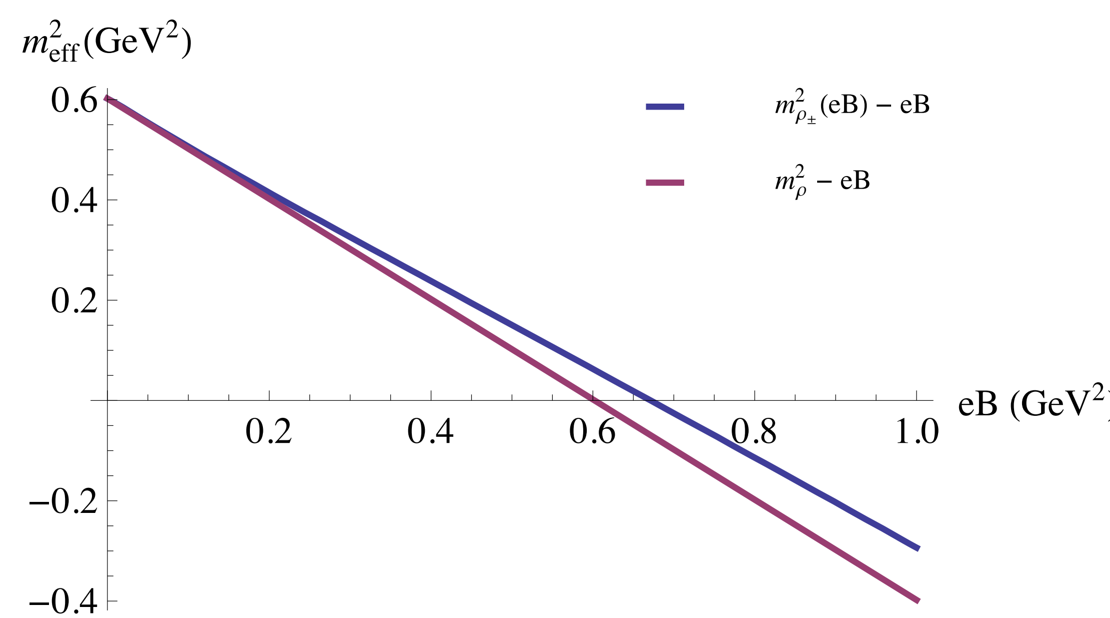
Extended NJL model

Chernodub,1101.0117



AdS/CFT models

Callebaut, Dudal, Verschelde, 1105.2217



Vafa-Witten theorem

No SSB occurs in the isospin channel.

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- **Fermion determinant is nonnegative.**

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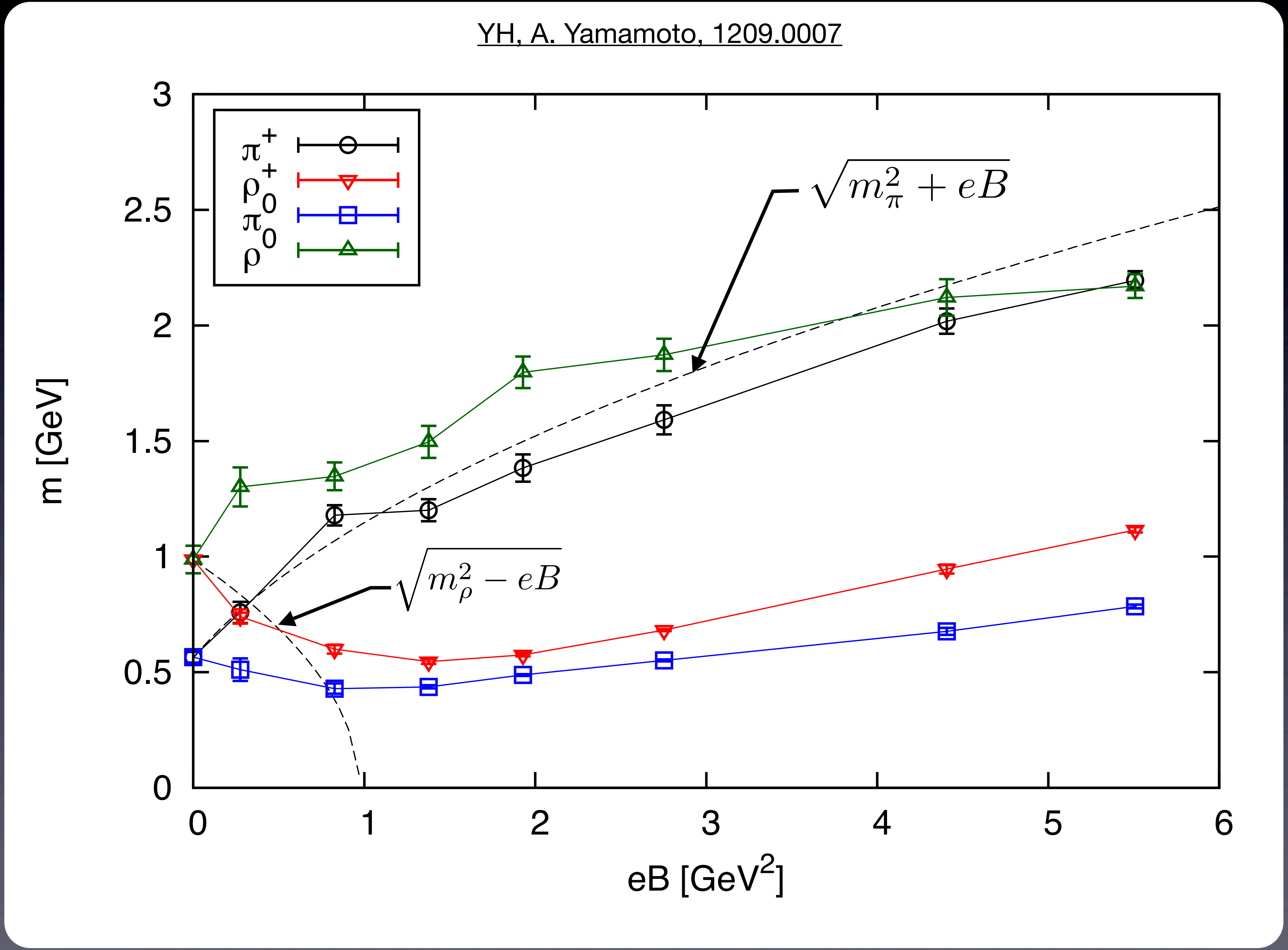
At finite B?

➡ No symmetry breaking.

Lattice study

YH, A. Yamamoto ('12)



Summary II

No vector meson condensation in QCD at finite *B*.

Vector meson mass degreases at small *B*. It increases at large *B*.

QCD inequality is useful tool to constrain effective models.

Comments
Does VW theorem work at

Finite T ?

Finite μ_B ?

Finite μ_I ?

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$\frac{1}{\not{D} + m + \gamma_4 \tau_3 \mu_I}$ can be zero.

Generalized NJL model?

$$\mathcal{L} = \bar{\psi}(\not{D} + m)\psi + \frac{1}{2G}V_\mu^2$$

$$D_\mu = \partial_\mu - i\tau^a V_\mu^a - iqA_\mu^{\text{em}}$$

Supersymmetric model?

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